

DYNAMICAL ELECTROWEAK SYMMETRY BREAKING AND THE TOP QUARK*

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ABSTRACT

In this talk I discuss theories of dynamical electroweak symmetry breaking, with emphasis on the implications of a heavy top-quark on the weak-interaction ρ parameter.

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1 What's Wrong with the Standard Model?

In the standard one-doublet Higgs model one introduces a fundamental scalar doublet of $SU(2)_W$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \quad (1.1)$$

which has a potential of the form

$$V(\phi) = \lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2 . \quad (1.2)$$

In the potential, v^2 is assumed to be positive in order to favor the generation of a non-zero vacuum expectation value for ϕ . This vacuum expectation value breaks the electroweak symmetry, giving mass to the W and Z .

This explanation of electroweak symmetry breaking is unsatisfactory for a number of reasons. For one thing, this model does not give a dynamical explanation of electroweak symmetry breaking. For another, when embedded in theories with additional dynamics at higher energy scales, these theories are technically unnatural².

Perhaps most unsatisfactory, however, is that theories of fundamental scalars are probably “trivial”,³ *i.e.*, it is not possible to construct an interacting theory of scalars in four dimensions that is valid to arbitrarily short distance scales. In quantum field theories, fluctuations in the vacuum screen charge – the vacuum acts as a dielectric medium. Therefore there is an effective coupling constant which depends on the energy scale (μ) at which it is measured. The variation of the coupling with scale is summarized by the β -function of the theory

$$\beta(\lambda) = \mu \frac{d\lambda}{d\mu} . \quad (1.3)$$

The only coupling in the Higgs sector of the standard model is the Higgs self-coupling λ . In perturbation theory, the β -function is calculated to be

$$\text{Diagram: a loop with two external lines} \rightarrow \beta = \frac{3\lambda^2}{2\pi^2} . \quad (1.4)$$

Using this β -function, one can compute the behavior of the coupling constant as a function of the scale*. One finds that the coupling at a scale μ is related to the coupling at some higher

*Since these expressions were computed in perturbation theory, they are only valid when $\lambda(\mu)$ is sufficiently small. For large coupling we must rely on non-perturbative lattice monte-carlo studies,^{4,5} which show behavior similar to that implied by the perturbative expressions derived here.

scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log \frac{\Lambda}{\mu} \quad . \quad (1.5)$$

In order for the Higgs potential to be stable, $\lambda(\Lambda)$ has to be positive. This implies

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log \frac{\Lambda}{\mu} \quad . \quad (1.6)$$

Thus, we have the bound

$$\lambda(\mu) \leq \frac{2\pi^2}{3 \log \left(\frac{\Lambda}{\mu} \right)} \quad . \quad (1.7)$$

If this theory is to make sense to arbitrarily short distances, and hence arbitrarily high energies, we should take Λ to ∞ while holding μ fixed at about 1 TeV. In this limit we see that the bound on λ goes to zero. In the continuum limit, this theory is trivial; it is free field theory.

The theory of a relatively light weakly coupled Higgs boson, can be self-consistent to a very high energy. For example, if the theory is to make sense up to a typical GUT scale energy, 10^{16} GeV, then the Higgs boson mass has to be less than about 170 GeV.⁶ In this sense, although a theory with a light Higgs boson does not really answer any of the interesting questions (*e.g.*, it does not explain *why* $SU(2)_W \times U(1)_Y$ breaking occurs), the theory does manage to postpone the issue up to higher energies.

2 Dynamical Electroweak Symmetry Breaking

2.1 Technicolor

Technicolor⁷ theories strive to explain electroweak symmetry breaking in terms of physics operating at an energy scale of order a TeV. In technicolor theories, electroweak symmetry breaking is the result of chiral symmetry breaking in an asymptotically-free, strongly-interacting gauge theory with massless fermions. Unlike theories with fundamental scalars, these theories are technically natural: just as the scale Λ_{QCD} arises in QCD by dimensional transmutation, so too does the weak scale v in technicolor theories. Accordingly, it can be exponentially smaller than the GUT or Planck scales. Furthermore, asymptotically-free non-abelian gauge theories may be fully consistent quantum field theories.

In the simplest theory⁷ one introduces doublet of new massless fermions

$$\Psi_L = \begin{pmatrix} U \\ D \end{pmatrix}_L \quad U_R, D_R \quad (2.8)$$

which are N 's of a technicolor gauge group $SU(N)_{TC}$. In the absence of electroweak interactions, the Lagrangian for this theory may be written

$$\mathcal{L} = \bar{U}_L i \not{D} U_L + \bar{U}_R i \not{D} U_R + \quad (2.9)$$

$$\bar{D}_L i \not{D} D_L + \bar{D}_R i \not{D} D_R \quad (2.10)$$

and thus has an $SU(2)_L \times SU(2)_R$ chiral symmetry. In analogy with QCD, we expect that when technicolor becomes strong,

$$\langle \bar{U}_L U_R \rangle = \langle \bar{D}_L D_R \rangle \neq 0, \quad (2.11)$$

which breaks the global chiral symmetry group down to $SU(2)_{L+R}$, the vector subgroup (analogous to isospin in QCD).

If we weakly gauge $SU(2) \times U(1)$, with the left-handed technifermions forming a weak doublet and identify hypercharge with a symmetry generated by a linear combination of the T_3 in $SU(2)_R$ and technifermion number, then chiral symmetry breaking will result in the electroweak gauge group's breaking down to electromagnetism. The Higgs mechanism then produces the appropriate masses for the W and Z bosons if the F -constant of the technicolor theory (the analog of f_π in QCD) is approximately 246 GeV. (The residual $SU(2)_{L+R}$ symmetry insures that, to lowest-order, $M_W = M_Z \cos \theta_W$ and the weak interaction ρ -parameter equals one at tree-level.⁸)

2.2 Top-Mode and Strong-ETC Models

There is also a class of theories in which the scale (M) of the dynamics responsible for (all or part of) electroweak symmetry breaking can, in principle, take any value of order a TeV or greater. These models, inspired by the Nambu–Jona-Lasinio (NJL) model⁹ of chiral symmetry breaking in QCD, involve a strong, but *spontaneously broken*, non-confining gauge interaction. Examples include top quark condensate (and related) models,^{10–14} as well as models with strong extended technicolor interactions.¹⁵ When the strength of the effective four-fermion interaction describing the broken gauge interactions – *i.e.* the strength of the extended technicolor interactions in strong ETC models or the strength of other gauge interactions in top-condensate models – is adjusted close to the critical value for chiral symmetry breaking, the high-energy dynamics may play a role in electroweak symmetry breaking without driving the electroweak scale to a value of order M .

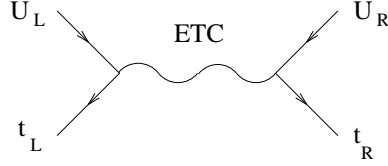
The high-energy dynamics must have the appropriate properties in order for it to play a role in electroweak symmetry breaking¹⁶: If the coupling constants of the high-energy theory are small, only low-energy dynamics (such as technicolor) can contribute to electroweak symmetry breaking. If the coupling constants of the high-energy theory are large and the interactions are attractive in the appropriate channels, chiral symmetry will be broken by the high-energy interactions and the scale of electroweak symmetry breaking will be of order M . If the transition between these two extremes is continuous, *i.e.* if the chiral symmetry breaking phase transition is *second order* in the high-energy couplings, then it is possible to adjust the high-energy parameters so that the dynamics at scale M can contribute to electroweak symmetry breaking. The adjustment of the high-energy couplings is a reflection of the fine-tuning required to create a hierarchy of scales.

What is crucial is that the transition be (at least approximately) second order in the high-energy couplings. If the transition is first order, then as one adjusts the high-energy couplings the scale of chiral symmetry breaking will jump discontinuously from approximately zero at weak coupling to approximately M at strong coupling. Therefore, if the transition is first order, it will generally not be possible to maintain any hierarchy between the scale of electroweak symmetry breaking and the scale of the high-energy dynamics.

If the transition is second order and if there is a *large* hierarchy of scales ($M \gg 1$ TeV), then close to the transition the theory may be described in terms of a low-energy effective Lagrangian with composite “Higgs” scalars – the Ginsburg-Landau theory of the chiral phase transition. However, if there is a large hierarchy, the arguments of triviality given in the first section apply to the effective low-energy Ginsburg-Landau theory describing the composite scalars: the effective low-energy theory would be one which describes a *weakly* coupled theory of (almost) fundamental scalars, despite the fact that the “fundamental” interactions are strongly self-coupled!

3 m_t in Models of Dynamical EWSB

In technicolor models, the masses of the ordinary fermions are due to their coupling to the technifermions, whose chiral-symmetry breaking is responsible for electroweak symmetry breaking. This is conventionally¹⁷ assumed to be due to additional, broken, extended-technicolor (ETC) gauge-interactions:



(3.1)

which leads to a mass for the top-quark

$$m_t \approx \frac{g^2}{M_{ETC}^2} \langle \bar{U}U \rangle_{M_{ETC}} , \quad (3.2)$$

where we have been careful to note that it is the value of the technifermion condensate renormalized at the scale M_{ETC} which is relevant.

For a QCD-like technicolor, there is no substantial difference between $\langle \bar{U}U \rangle_{M_{ETC}}$ and $\langle \bar{U}U \rangle_{\Lambda_{TC}}$, and we can use naive dimensional analysis¹⁸ to estimate the technifermion condensate, arriving at a top-quark mass

$$m_t \approx \frac{g^2}{M_{ETC}^2} 4\pi F^3 , \quad (3.3)$$

We can invert this relation to find the characteristic mass-scale of top-quark mass-generation

$$\frac{M_{ETC}}{g} \approx 1 \text{ TeV} \left(\frac{F}{246 \text{ GeV}} \right)^{\frac{3}{2}} \left(\frac{175 \text{ GeV}}{m_t} \right)^{\frac{1}{2}} . \quad (3.4)$$

We immediately see that the scale of top-quark mass generation is likely to be *quite* low, unless the value of the technifermion condensate ($\langle \bar{U}U \rangle_{M_{ETC}}$) can be raised significantly above the value predicted by naive dimensional analysis. The prospect of such a low ETC-scale is both tantalizing and problematic. As we will see in the next section, constraints from the deviation of the weak interaction ρ parameter from one suggest that the scale may have to be larger than one TeV.

There have been two approaches to enhance the technifermion condensate which have been discussed in the literature: “walking”¹⁹ and “strong-ETC”.¹⁵ In a walking theory, one arranges for the technicolor coupling constant to be approximately constant and large over some range of momenta. The maximum enhancement that one might expect in this scenario is

$$\langle \bar{U}U \rangle_{M_{ETC}} \approx \langle \bar{U}U \rangle_{\Lambda_{TC}} \left(\frac{M_{ETC}}{\Lambda_{TC}} \right)^{\gamma(\alpha_{TC})} , \quad (3.5)$$

where $\gamma(\alpha_{TC})$ is the anomalous dimension of the technifermion mass operator (which is possibly as large as one). As described above, however, we expect that M_{ETC} cannot be too much higher than Λ_{TC} , and therefore that the enhancement due to walking is not sufficient to reconcile the top-quark mass and an ETC scale higher than a TeV.

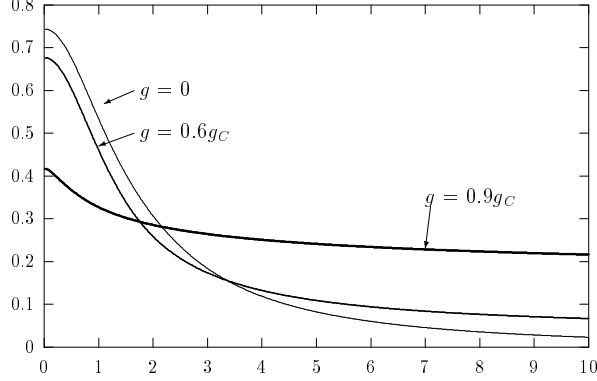


Figure 1: Plot²⁰ of technifermion self energy vs. momentum (both measured in TeV), as predicted by the gap-equation in the rainbow approximation, for various strengths of the ETC coupling relative to their critical value g_C .

The strong-ETC alternative is potentially more promising. As the size of the ETC-coupling at the ETC scale approaches the critical value for chiral symmetry breaking, it is possible to enhance the running technifermion self-energy $\Sigma(k)$ at large momenta (see Fig. 1). Since the technifermion condensate is related to the trace of the fermion propagator.

$$\langle \bar{U}U \rangle_{M_{ETC}} \propto \int^{M_{ETC}^2} dk^2 \Sigma(k) , \quad (3.6)$$

a slowly-falling running-mass translates to an enhanced condensate[†].

Unfortunately, there is no such thing as a free lunch. As we see from Fig. 2, the enhancement of the technifermion self-energy in strong-ETC theories comes at the cost of a “fine-tuning” of the strength of the ETC coupling relative to the critical value where the ETC interactions would, in and of themselves, generate chiral symmetry breaking. In the context of the NJL approximation, we find that enhancement of the top quark mass is directly related to the

[†]More physically, in terms of the relevant low-energy theory, it can be shown that the enhancement of the top-quark mass is due to the dynamical generation of a light scalar state^{16,21}

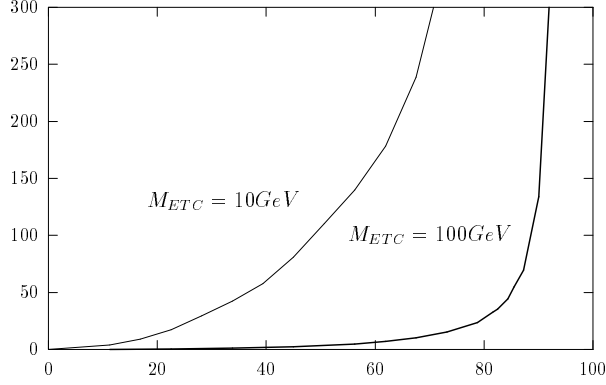


Figure 2: Plot²⁰ of top mass (in GeV) vs. ETC coupling (g/g_C in %), as predicted by gap-equation in the rainbow approximation, for ETC scales of 10 and 100 TeV.

severity of this adjustment. In particular, if we denote the critical value of the ETC coupling by g_C , in the NJL approximation we find⁹

$$\frac{\langle \bar{U}U \rangle_{\Lambda_{TC}}}{\langle \bar{U}U \rangle_{M_{ETC}}} \approx \frac{\Delta g^2}{g_C^2} \quad (3.7)$$

where $\Delta g^2 \equiv g^2 - g_C^2$.

4 $\Delta\rho_*$

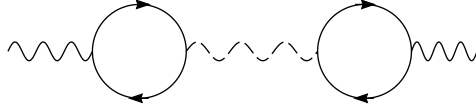
The physics which is responsible for top-quark mass generation must violate custodial $SU(2)$ since, after all, this physics must give rise to the disparate top- and bottom-quark masses. The danger is that this isospin violation will “leak” into the W and Z gauge-boson masses and to give rise to a deviation of the weak interaction ρ -parameter from one.

4.1 Direct Contributions

As emphasized by Appelquist, Bowick, Cohler, and Hauser,²² ETC operators which violate custodial isospin by two units ($\Delta I = 2$) are particularly dangerous. Denoting the right-handed technifermion doublet by Ψ_R , consider the operator

$$\frac{g^2}{M^2} (\bar{\Psi}_R \gamma_\mu \sigma_3 \Psi_R)^2 \quad , \quad (4.1)$$

which can result in the (mass-)mixing of the Z with an isosinglet ETC gauge-boson



(4.2)

and hence a contribution to $\Delta\rho$. Contributions of this sort arise naturally in ETC-models which give rise to the top-quark mass.²⁴

If there are N_D doublets of the technifermions Ψ , and they give rise to a contribution to M_W^2 proportional to $N_D F^2$, the contribution of the operator in eqn. (4.1) to the ρ parameter can be estimated to be

$$\Delta\rho_* \approx \frac{2g^2}{M^2} \frac{N_D^2 F^4}{v^2} \quad (4.3)$$

$$\approx 12\% g^2 \left(\frac{N_D F^2}{(246 \text{ GeV})^2} \right)^2 \left(\frac{1 \text{ TeV}}{M} \right)^2. \quad (4.4)$$

Current limits (see Fig. 3) on the parameter T ($\Delta\rho_* = \alpha T$) imply that $\Delta\rho_* \lesssim 0.4\%$.

There are two ways[‡] in which one may try to satisfy this constraint. The equation above implies

$$\frac{M}{g} \gtrsim 5.5 \text{ TeV} \left(\frac{N_D F^2}{(246 \text{ GeV})^2} \right). \quad (4.5)$$

If $N_D F^2 \approx (246 \text{ GeV})^2$, that is if the sector giving rise to the top-quark mass is responsible for the bulk of EWSB, then the scale M must be much larger than the naive 1 TeV expectation in QCD-like technicolor. Comparing this with eqns. (3.4) and (3.7) above, we see that the enhancement of the condensate needed requires a fine-tuning of order 3% ($\approx (1/5.5)^2$) in order to produce a top-quark mass of order 175 GeV.

Alternatively, we may re-write the bound as

$$F \lesssim \frac{105 \text{ GeV}}{\sqrt{N_D}} \left(\frac{M/g}{1 \text{ TeV}} \right)^{\frac{1}{2}} \quad (4.6)$$

[‡]It is also conceivable²⁸ that there are additional isospin-asymmetric contributions — say, from relatively light pseudo-Goldstone bosons — which give rise to *negative* contributions to T and cancel some or all of the positive contributions discussed here.

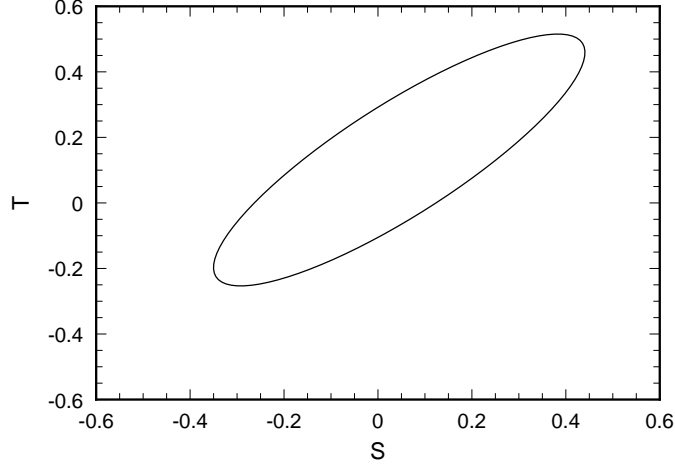


Figure 3: The ellipse²³ in the $S - T$ plane which projects onto the 95% confidence range for T . Note that $\Delta\rho_* = \alpha T$.

If M/g is of order 1 TeV, it is necessary that the sector responsible for top quark mass generation *not* give rise to the bulk of EWSB. While this case is counter-intuitive (after all, the third generation is the heaviest!), it may in fact provide a resolution to the issue of how large isospin breaking can exist in the fermion mass spectrum without leaking into the W and Z masses. This is essentially what happens in multiscale models^{25,26} and in top-color assisted technicolor.²⁷ Such hierarchies of technifermion masses are also useful for reducing the predicted value of S in technicolor models^{§,28}

4.2 Indirect Contributions

A second class of potentially dangerous contributions come from isospin violation in the technifermion mass spectra. In a manner analogous to the contribution²⁹ of the $t - b$ mass splitting to $\Delta\rho$, any difference in the dynamical masses of two technifermions in the *same* doublet will give rise to deviations in the ρ parameter from one. The size of this effect can be estimated a la Pagels-Stokar.³⁰ Using this approximation, we find that the contributions to the loop diagram from low-momenta dominate and

[§]Recently the experimental upper bound on S has been relaxed, so that positive values of S are allowed ($S < 0.4$ at the 95% confidence level).²³

$$\Delta\rho_* \propto \frac{N_D d}{16\pi^2} \left(\frac{\Sigma_U(0) - \Sigma_D(0)}{v} \right)^2 \quad (4.7)$$

where N_D and d are the the number of doublets and dimension of the technicolor representation respectively. Since we require $\Delta\rho_* \lesssim 0.4\%$, the equation above implies

$$N_D d \left(\frac{\Delta\Sigma(0)}{m_t} \right)^2 \lesssim 1.3 . \quad (4.8)$$

From this we see that, $\Delta\Sigma(0)$ must be less than of order m_t (perhaps, given the crude approximations involved, one may be able to live with $d = 2$ in the fundamental of and $SU(2)$ technicolor group with one doublet).

However, if the t and b get their mass from the same technidoublet, then at the ETC-scale we expect that there is no difference between the t , b and the corresponding technifermions¹⁵

$$\begin{aligned} \Delta\Sigma(M_{ETC}) &\equiv \Sigma_U(M_{ETC}) - \Sigma_D(M_{ETC}) \approx \\ \Delta m(M_{ETC}) &\equiv m_t(M_{ETC}) - m_b(M_{ETC}) . \end{aligned} \quad (4.9)$$

Furthermore, if QCD is the only interaction which contributes to the scaling of the t and b masses, we expect $\Delta m(M_{ETC}) \approx m_t^{pole}$, and from scaling properties of the technifermion self-energies, we expect $\Delta\Sigma(0) \gtrsim \Delta\Sigma(M_{ETC})$.

There are two ways to avoid these constraints. One is that perhaps there are *additional* interactions which contribute to the scaling of the top- and bottom-masses below the ETC scale, and hence that $\Delta m(M_{ETC}) \ll m_t^{pole}$. This would be the case if the t and/or b get only a *portion* of their mass from the technicolor interactions, and would imply that the third generation must have (strong) interactions different from the technifermions (and possibly from the first and second generations). Another possibility is that the t and b get mass from *different* technidoublets, each of which have isospin-symmetric masses. The first alternative is the solution chosen in top-color assisted technicolor models (see below), while the latter has only recently begun to be explored.³¹

5 Case Study: Top-Color Assisted Technicolor

Recently, Hill has combined aspects of two different approaches to dynamical electroweak symmetry breaking into a model which he refers to as top-color assisted technicolor.²⁷ In this model a top-condensate is driven by the combination of a strong, but spontaneously broken

and non-confining, isospin-symmetric top-color interaction and an additional (either weak or strong) isospin-breaking $U(1)$ interaction which couple only to the third generation quarks.

At low-energies, the top-color and hypercharge interactions of the third generation quarks may be approximated by four-fermion operators.²⁷

$$\mathcal{L}_{4f} = -\frac{4\pi\kappa_{tc}}{M^2} \left[\overline{\psi}\gamma_\mu \frac{\lambda^a}{2} \psi \right]^2 - \frac{4\pi\kappa_1}{M^2} \left[\frac{1}{3} \overline{\psi}_L \gamma_\mu \psi_L + \frac{4}{3} \overline{t}_R \gamma_\mu t_R - \frac{2}{3} \overline{b}_R \gamma_\mu b_R \right]^2, \quad (5.1)$$

where ψ represents the top-bottom doublet, κ_{tc} and κ_1 are related respectively to the top-color and $U(1)$ gauge-couplings squared, and where (for convenience) we have assumed that the top-color and $U(1)$ gauge-boson masses are comparable and of order M . The first term in equation (5.1) arises from the exchange of top-color gauge bosons, while the second term arises from the exchange of the new $U(1)$ hypercharge gauge boson which has couplings proportional to the ordinary hypercharge couplings. In order to produce a large top quark mass without giving rise to a correspondingly large bottom quark mass, the combination of the top-color and extra hypercharge interactions are assumed to be critical in the case of the top quark but not the bottom quark. The criticality condition for top quark condensation in this model is then:

$$\kappa_{eff}^t = \kappa_{tc} + \frac{1}{3}\kappa_1 > \kappa_c = \frac{3\pi}{8} > \kappa_{eff}^b = \kappa_{tc} - \frac{1}{6}\kappa_1. \quad (5.2)$$

The contribution of the top-color sector to electroweak symmetry breaking can be quantified by the F-constant of this sector. In the NJL approximation,⁹ for M of order 1 TeV, and $m_t \approx 175$ GeV, we find

$$f_t^2 \equiv \frac{N_c}{8\pi^2} m_t^2 \log \left(\frac{M^2}{m_t^2} \right) \approx (64 \text{ GeV})^2. \quad (5.3)$$

As f_t is small compared to 246 GeV, there must be additional dynamics which is largely responsible for giving rise to the W and Z masses. In top-color assisted technicolor, technicolor interactions play that role.

5.1 Direct Isospin Violation

Technifermions are necessary to produce the bulk of EWSB and to give mass to the light fermions. However, the heavy and light fermions must mix — hence, we would naturally expect that at least some of the *technifermions* carry the extra $U(1)$ interaction. If the additional $U(1)$ interactions violate custodial symmetry[¶], the $U(1)$ coupling will have to be quite small to keep

[¶]It has been noted³¹ that if the top- and bottom-quarks receive their masses from *different* technidoublets, it is possible to assign the extra $U(1)$ quantum numbers in a custodially invariant fashion.

this contribution to $\Delta\rho_*$ small.²³ We will illustrate this in the one-family technicolor³² model, assuming that techniquarks and technileptons carry $U(1)$ -charges proportional to the hypercharge of the corresponding ordinary fermion^{||}. We can rewrite the effective $U(1)$ interaction of the technifermions as

$$\mathcal{L}_{4T1} = -\frac{4\pi\kappa_1}{M^2} \left[\frac{1}{3} \bar{\Psi} \gamma_\mu \Psi + \bar{\Psi}_R \gamma_\mu \sigma^3 \Psi_R - \bar{L} \gamma_\mu L + \bar{L}_R \gamma_\mu \sigma^3 L_R \right]^2, \quad (5.4)$$

where Ψ and L are the techniquark and technilepton doublets respectively.

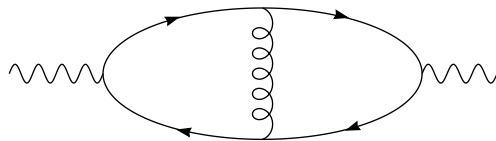
From the analysis given above (eqn. (4.4)), we see that the contribution to $\Delta\rho_*$ from degenerate technifermions is²³:

$$\Delta\rho_*^T \approx 152\% \kappa_1 \left(\frac{1 \text{ TeV}}{M} \right)^2. \quad (5.5)$$

Therefore, if M is of order 1 TeV and the extra $U(1)$ has isospin-violating couplings to technifermions, κ_1 must be extremely small.

5.2 Indirect Isospin Violation

In principle, since the isospin-splitting of the top and bottom are driven by the combination of top-color and the extra $U(1)$, the technifermions can be degenerate. In this case, the only indirect contribution to the ρ parameter at one-loop is the usual contribution coming from loops of top- and bottom-quarks.²⁹ However, since there are additional interactions felt by the third-generation of quarks, there are “two-loop” contributions of the form



(5.6)

This contribution yields²³

$$\Delta\rho_*^{\text{tc}} \approx 0.53\% \left(\frac{\kappa_{tc}}{\kappa_c} \right) \left(\frac{1 \text{ TeV}}{M} \right)^2 \left(\frac{f_t}{64 \text{ GeV}} \right)^4. \quad (5.7)$$

Combining this with eqn. (5.3), we find that

$$M \gtrsim 1.4 \text{ TeV} \quad (5.8)$$

^{||}Note that this choice is anomaly-free.

for $\kappa_{tc} \approx \kappa_c$.

This immediately puts a constraint on the mass of the top-color gluon which is comparable to the direct limits currently obtained by CDF.³³

5.3 Fine-Tuning

Finally, we must require that the sum of the effects of eqns. (5.5) and (5.7) do not give rise to an experimentally disallowed contribution to the ρ parameter. Equation (5.5) implies that κ_1 must either be very small, or M very large. However, we must also simultaneously satisfy the constraint of eqn. (5.2), which implies that

$$\frac{\Delta\kappa_{tc}}{\kappa_c} = \left| \frac{\kappa_{tc} - \kappa_c}{\kappa_c} \right| \leq \frac{1}{3} \frac{\kappa_1}{\kappa_c} , \quad (5.9)$$

Therefore, if M is low and κ_1 is small, the top-color coupling must be tuned close to the critical value for chiral symmetry breaking. On the other hand, if κ_1 is not small and M is relatively large the *total* coupling of the top-quark must be tuned close to the critical NJL value for chiral symmetry breaking in order to keep the top-quark mass low. The gap-equation for the Nambu–Jona-Lasinio model implies that

$$\frac{\Delta\kappa_{eff}}{\kappa_c} = \frac{\kappa_{eff}^t - \kappa_c}{\kappa_c} = \frac{\frac{m_t^2}{M^2} \log \frac{M^2}{m_t^2}}{1 - \frac{m_t^2}{M^2} \log \frac{M^2}{m_t^2}} . \quad (5.10)$$

These two constraints are shown in Fig. 4. For $M > 1.4$ TeV, we find that either $\Delta\kappa_{tc}/\kappa_c$ or $\Delta\kappa_{eff}/\kappa_c$ must be tuned to less than 1%. This trade-off in fine tunings is displayed in figure 4. For the “best” case where both tunings are of order 1%, $M = 4.5$ TeV.

6 Conclusions

We have seen that a large top quark mass has a number of important implications for dynamical electroweak symmetry breaking:

- A large top-quark mass naturally implies, in models of dynamical electroweak symmetry breaking, the possibility of a correspondingly low scale for the scale of top flavor-physics. While I have emphasized the constraints on such physics arising from potential contributions to the weak interaction ρ parameter, there are also significant constraints arising from the size of the $Z \rightarrow b\bar{b}$ branching ratio,^{34,24} as well as from contributions to $b \rightarrow s\gamma$ and $B - \bar{B}$ mixing.^{35–37}

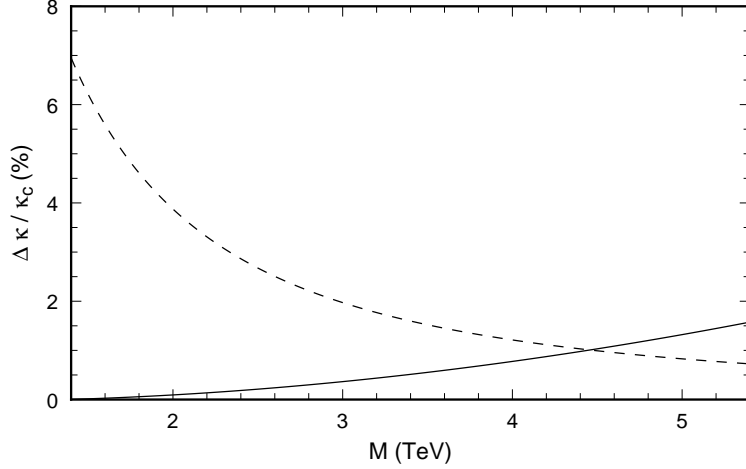


Figure 4: The amount of fine-tuning required²³ in the TC^2 model. The dashed line is the amount of fine-tuning in $\Delta\kappa_{eff}$ required to keep m_t much lighter than M , see equation (5.10). The solid curve shows the amount of fine-tuning (see equation (5.9)) in $\Delta\kappa_{tc}$ required to satisfy the bound $\Delta\rho_* < 0.4\%$. The region excluded by the experimental constraint on $\Delta\rho_*$ is above the solid curve.

- The physics responsible for the large isospin breaking in the $t - b$ mass splitting can lead to potentially dangerous “direct” and “indirect” effects in the W and Z masses.
- The direct and indirect effects can be mitigated if the sector which is responsible for the top- and bottom-masses does *not* provide the bulk of electroweak symmetry breaking and, conversely, if the sector responsible for the W and Z masses gives rise to only a *small portion* of the top- and bottom-masses. This can happen only if the top and bottom feel *strong* interactions which are not shared by the technifermions and, possibly, the first two generations.
- In top-color assisted technicolor, the extra top-color interactions give rise to additional indirect contributions to $\Delta\rho$, and we must require that $M_g \gtrsim 1.4$ TeV. Furthermore, If the extra $U(1)$ has isospin-violating couplings to technifermions, we require fine-tuning of order 1%.

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